wjec cbac

# GCE AS/A level

0977/01

S16-0977-01

# MATHEMATICS – FP1
## Further Pure Mathematics

A.M. FRIDAY, 24 June 2016

1 hour 30 minutes

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** Differentiate $\dfrac{x^2}{x+1}$ from first principles. [7]

**2.** The transformation *T* in the plane consists of an anticlockwise rotation through 90° about the origin followed by a translation in which the point $(x, y)$ is transformed to the point $(x + 1, y + 2)$.

*(a)* Determine the 3 × 3 matrix which represents *T*. [4]

*(b)* Find the fixed point of *T*. [4]

**3.** Given that

$$S_n = \sum_{r=1}^{n} r^2(r+1),$$

obtain an expression for $S_n$ in terms of $n$, giving your answer as a product of four linear factors. [6]

**4.** The complex numbers $z_1$, $z_2$ are given by

$$z_1 = -\sqrt{3} + \mathrm{i};\ \ z_2 = 1 + \mathrm{i}.$$

*(a)* Determine the modulus and the argument of each of $z_1$, $z_2$, giving **exact** values of the moduli and giving the arguments in terms of $\pi$. [4]

*(b)* The complex number $w$ is given by

$$w = \frac{z_1^{\,2}}{z_2}.$$

Using your results from *(a)*, or otherwise, determine $w$ in the form $a + \mathrm{i}b$, giving $a$, $b$ correct to two decimal places. [6]

**5.** The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} 2 & 5 & \lambda \\ 0 & \lambda & -1 \\ \lambda & 2 & 1 \end{bmatrix}.$$

*(a)* (i) Show that

$$\det \mathbf{M} = 4 - 3\lambda - \lambda^3.$$

(ii) Hence show that $\mathbf{M}$ is singular when $\lambda = 1$ and is not singular for any other real values of $\lambda$.

(iii) Show that the following system of equations is consistent and find the general solution. [12]

$$\begin{bmatrix} 2 & 5 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

*(b)* Suppose now that $\lambda = -1$. By first finding the adjugate matrix of $\mathbf{M}$, determine the inverse matrix $\mathbf{M}^{-1}$. [5]

**6.** Consider the cubic equation

$$ax^3 + bx^2 + cx + d = 0.$$

Given that the product of two of the roots is equal to 1, show that

$$d^2 - bd = a^2 - ac.$$ [6]

**7.** The sequence $x_1, x_2, x_3, \ldots$ is generated by the relationship

$$x_{n+1} = 2x_n - n + 1 \quad \text{where} \quad x_1 = 3.$$

Use mathematical induction to prove that

$$x_n = 2^n + n$$

for all positive integers $n$. [6]

**8.** The function $f$ is defined on the domain $\left(0, \frac{\pi}{2}\right)$ by

$$f(x) = x^{\sin x}.$$

*(a)* Obtain an expression for $f'(x)$. [4]

*(b)* Given that the graph of $f$ has one stationary point, show that its $x$-coordinate lies between 0·35 and 0·36. [3]

**TURN OVER.**

**9.** The complex numbers $z$ and $w$ are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = (z + 2\mathrm{i})^2.$$

*(a)* Obtain expressions for $u$ and $v$ in terms of $x$ and $y$. [4]

*(b)* The point $P$ moves along the line $y = x - 1$. Find the equation of the locus of $Q$. [4]

**END OF PAPER**